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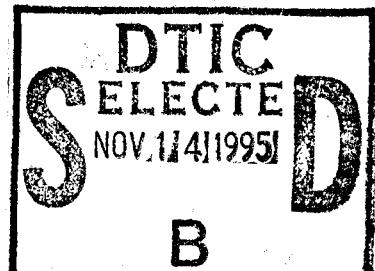
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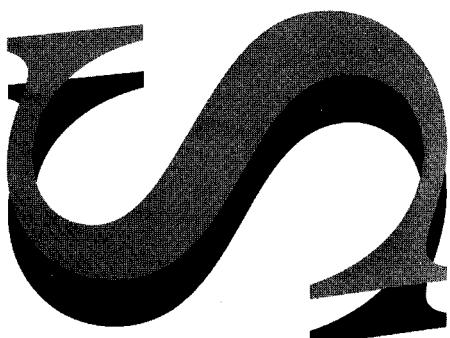
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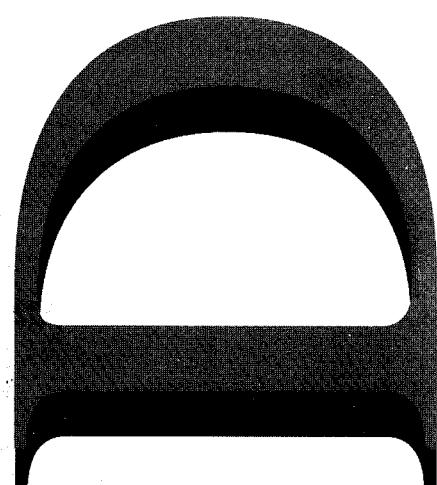
Input Impedance Calculation for  
TEM Horn Antennas

Geoff Staines and Sean Braidwood



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# Input Impedance Calculation for TEM Horn Antennas

*Geoff Staines and Sean Braidwood*

**Electronic Warfare Division  
Electronics and Surveillance Research Laboratory**

DSTO-TR-0187

## ABSTRACT

This report describes a technique for calculating the input impedance of ideal, infinitely long TEM horn antennas used to radiate ultrawideband signals. A simple potential calculation is used to determine the fields of the TEM mode of the antenna, from which the input impedance is derived. This result is useful for matching TEM horns to the output impedance of the excitation source to avoid undesirable reflected signals.

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# Input Impedance Calculation for TEM Horn Antennas

## EXECUTIVE SUMMARY

A simple numerical technique is described for calculating the input impedance of TEM horn antennas for ultrawideband (UWB) applications. UWB antennas are required to radiate subnanosecond pulses with minimal distortion. The required bandwidth is from a few hundred megahertz up to 10 gigahertz. TEM horn antennas have emerged as potential candidates for directional UWB applications such as impulse radar. The input impedance of the horn is typically matched to the output impedance of the source. At the DSTO Wideband Test Facility, the antenna geometry has previously been varied empirically to produce the required input impedance due to the unreliability of alternative theoretical techniques based on conformal mapping which was not corrected until recently. This simple analysis circumvents this time consuming process and is readily implemented.

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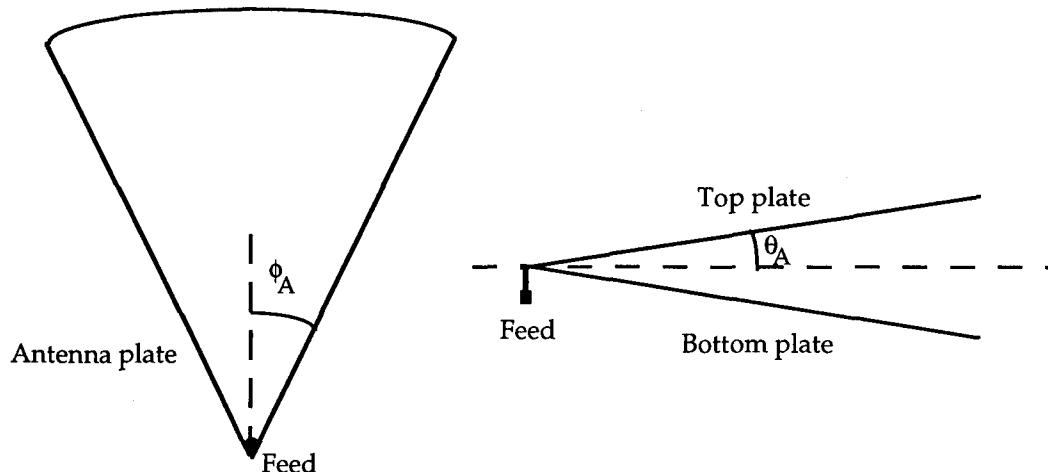
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## 1. Introduction

Recently there has been a growing interest in the possible defence applications of ultrawideband (UWB) radiation. These applications are currently being investigated at the DSTO Wideband Test Facility. The generation and transmission of these ultrawideband emissions such as impulse radiation, requires a fast electrical switch to generate a subnanosecond transient which directly excites a wideband antenna. If the excitation pulse, or more commonly its time derivative, is to be faithfully radiated in the far field zone, then a nondispersive antenna is required. This last requirement is a significant restriction, and rules out log-periodic antennas commonly used as compact, wideband antennas.

One class of antenna that is both wideband and nondispersive is the TEM horn, as shown in Figure 1.1. Two quasi-triangular plates are connected together at one vertex to form the feed. A spherical TEM wave propagates along the antenna structure, confined mainly between the plates. When the TEM mode reaches the end of the antenna structure, conversion to higher order propagating spherical TM and TE modes occurs because the TEM mode can no longer be supported. This type of antenna has been shown in the past to faithfully transmit the time derivative of the incident pulse in the far field zone [1].



*Figure 1.1: Conventional TEM horn geometry*

The input impedance of TEM horns is dependent both on the flare angle  $\theta_A$  and the apex angle of the plates,  $\phi_A$ . For this analysis, the length of the horn is assumed to be much longer than the spatial width of the excitation pulse, so that reflections from the end of the horn can be ignored. For the 100 ps pulses normally used in the DSTO

Wideband Test Facility, the spatial pulse width is 3 cm. This is small compared to the TEM horn antenna plates which are usually 1 m long. Therefore, the input impedance of the horn is the same as for an ideal infinitely long antenna for frequencies of practical interest. Note that the impedance of an antenna which supports only the TEM mode is independent of frequency. The input impedance of the antenna is generally required to match that of the voltage source (typically  $50\ \Omega$ ). Previously, this match has been obtained empirically. This report describes a simple analysis based on a finite difference approach to calculate the TEM mode electric field of an ideal infinitely long horn. From the calculated field distribution for given angles  $\theta_A$  and  $\phi_A$ , the impedance is calculated from the power transmitted through the antenna.

Usually,  $\phi_A$  is given and  $\theta_A$  is to be found based on the desired antenna impedance. Note that as  $\phi_A$  increases the beamwidth in the horizontal plane increases. To maintain the matching to the source impedance,  $\theta_A$  must increase. This has the effect of decreasing the beamwidth in the vertical plane as  $\phi_A$  increases. Antenna beamwidths are not predicted by this analysis, but the ability to calculate the value of  $\theta_A$  required to match the antenna to the source impedance for a given  $\phi_A$  removes one degree of freedom from the horn design. This considerably simplifies the design process.

Carrel [2] was the first to develop an analysis intended to calculate the input impedance of infinitely long TEM horn antennas. This technique relied upon conformal transformations to yield a simple analytic form for the impedance. Unfortunately, Carrel's analysis has been found to be incorrect due to an error in applying the conformal transformations. Lambert *et al* [3] have recently produced a revised version of Carrel's analysis which has been shown to yield the correct results. Note that until Lambert *et al* developed a corrected conformal mapping analysis, this technique was considered too unreliable to use. This report presents a simple two-dimensional analysis based on a finite-difference technique which does not require conformal mapping and is particularly simple to implement. In addition, antennas with arbitrary cross-sections in the  $(\theta, \phi)$  plane can be analysed since the technique is two-dimensional. This work arose from the application of a finite-difference time-domain analysis for ultrawideband antenna modelling [4]. The results of this finite-difference technique are in good agreement with experimentally measured input impedance values for several TEM horns.

## 2. Theory

### 2.1 The Finite Difference Calculation

From Collin [5], a TEM mode electric field in spherical geometry can be related to scalar functions  $g$  and  $\Phi$  by

$$\mathbf{E} = g(r)\nabla_t \Phi$$

where  $\Phi(\theta, \phi)$  satisfies

$$\nabla_t^2 \Phi = 0$$

and  $g(r)$  follows

$$\frac{d^2 g}{dr^2} + k^2 g(r) = 0$$

with  $k = \omega\sqrt{\mu_0\epsilon_0}$ . Therefore

$$g(r) = e^{-jkr}$$

and

$$\left[ \sin^2 \theta \frac{\partial^2}{\partial \theta^2} + \sin \theta \cos \theta \frac{\partial}{\partial \theta} + \frac{\partial^2}{\partial \phi^2} \right] \Phi(\theta, \phi) = 0 \quad (1)$$

Note that Equation 1 does not contain any dependence on  $k$  and hence frequency. Therefore, this analysis applies directly in either the frequency or time domain. The potential  $\Phi(\theta, \phi)$  can be represented as  $\Phi(i, j)$  on a spherical finite difference mesh discretised in  $\theta$  and  $\phi$  using

$$\theta_i = \frac{\pi}{2} - i \Delta_\theta \quad \text{and} \quad \phi_j = \frac{\pi}{2} + j \Delta_\phi$$

where

$$\Delta_\theta = \frac{\pi}{2(N_\theta+1)} \quad \text{and} \quad \Delta_\phi = \frac{\pi}{N_\phi}$$

for  $i = 0, 1, \dots, N_\theta$ , and  $j = 0, 1, \dots, N_\phi$ . Symmetry allows the entire antenna to be analysed by considering only the quadrant  $\theta = 0$  to  $\pi/2$  and  $\phi = \pi/2$  to  $3\pi/2$ . The spherical coordinate system is shown in Figure 2.1.

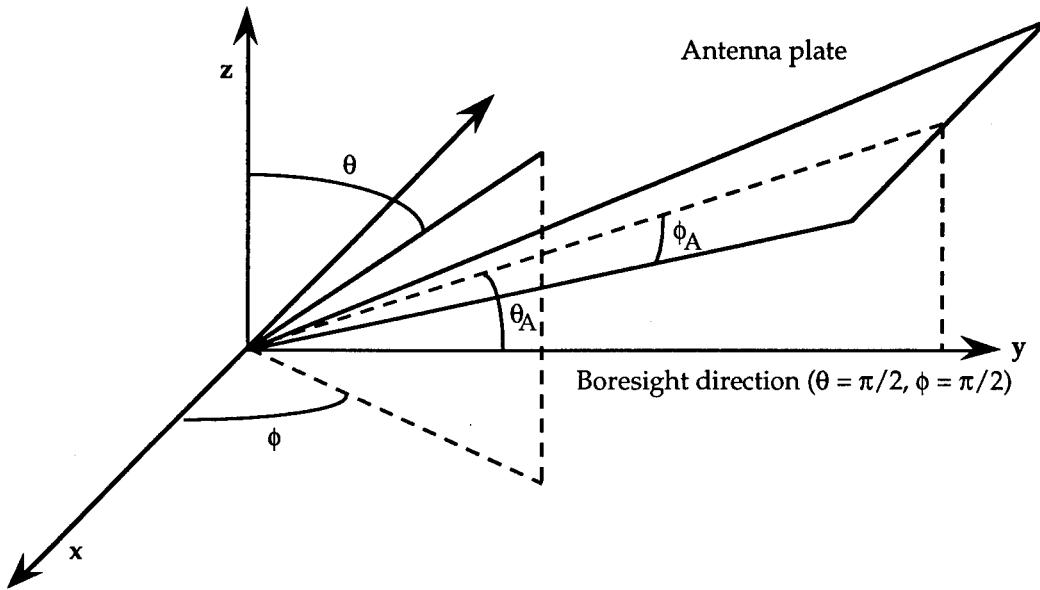


Figure 2.1: Coordinate system for TEM horn input impedance analysis

The antenna plate was assumed to have zero thickness and extend over  $\phi$  from  $j = 0$  to  $j = j_{plate}$  at the theta value corresponding to  $i = i_{plate}$ . This requirement for discretisation for the finite difference representation of Equation 1 meant that the flare and apex plate angles of the antenna,  $\theta_A$  and  $\phi_A$ , were rounded off to the nearest  $(\theta_i, \phi_j)$  values on the mesh. The effect of this rounding could be minimised by a judicious choice of  $N_\theta$  and  $N_\phi$ .

Expressing Equation 1 in its finite difference form leads to

$$\begin{aligned} \Phi(i+1,j) \frac{\sin \theta}{\Delta \theta} \left[ \frac{\sin \theta}{\Delta \theta} + \frac{\cos \theta}{2} \right] + \Phi(i-1,j) \frac{\sin \theta}{\Delta \theta} \left[ \frac{\sin \theta}{\Delta \theta} - \frac{\cos \theta}{2} \right] - 2 \Phi(i,j) \left[ \frac{\sin^2 \theta}{2} + \frac{1}{2} \right] \\ + \Phi(i,j+1) \frac{1}{\Delta \phi} + \Phi(i,j-1) \frac{1}{\Delta \phi} = 0 \end{aligned} \quad (2)$$

This can be written as

$$\mathbf{A} \Phi = \mathbf{b} \quad (3)$$

where

$$\Phi_k = \Phi(i,j)$$

$$\begin{aligned}
 \mathbf{A}_{kk} &= -2 \left[ \frac{\sin^2 \theta}{\Delta_\theta^2} + \frac{1}{\Delta_\phi^2} \right] \\
 \mathbf{A}_{k(k-1)} &= \mathbf{A}_{k(k+1)} = \frac{1}{\Delta_\phi} \\
 \mathbf{A}_{k(k+j_{max})} &= \frac{\sin \theta}{\Delta_\theta} \left[ \frac{\sin \theta}{\Delta_\theta} + \frac{\cos \theta}{2} \right] \\
 \mathbf{A}_{k(k-j_{max})} &= \frac{\sin \theta}{\Delta_\theta} \left[ \frac{\sin \theta}{\Delta_\theta} - \frac{\cos \theta}{2} \right]
 \end{aligned}$$

otherwise

$$\mathbf{A}_{kl} = 0$$

for  $k = i(N_\phi + 1) + j$ . For free space,  $\mathbf{b}_k = 0$  for all  $k$ , however  $\mathbf{A}_{kl}$  and  $\mathbf{b}_k$  will also include the boundary conditions which are described in the next section.

Equation 3 can be solved using successive overrelaxation based on a Gauss-Seidel iterative algorithm. An initial estimate of the potential  $\Phi(i,j)$  is made for all  $i, j$ . The value for the potential at each grid point is updated using

$$\Phi_k^{(m+1)} = (1-\omega) \Phi_k^{(m+1)} - \omega \frac{\sum_{l < k} \mathbf{A}_{kl} \Phi_l^{(m)} + \sum_{l > k} \mathbf{A}_{kl} \Phi_l^{(m)} - \mathbf{b}_k}{\mathbf{A}_{kk}}$$

The iteration was halted when the average magnitude of the correction to the potential over the entire mesh was less than  $10^{-5}$  typically. This condition was used to specify adequate convergence.

The relaxation parameter  $\omega$  is given by

$$\omega = \frac{2}{1 + \sqrt{1-\mu^2}}$$

where  $\mu$  is the spectral radius of the block Jacobi matrix  $\mathbf{B} = -\mathbf{D}^{-1}(\mathbf{L} + \mathbf{U})$  corresponding to  $\mathbf{A} = \mathbf{D} + \mathbf{L} + \mathbf{U}$  [6].  $\mathbf{D}$ ,  $\mathbf{L}$ , and  $\mathbf{U}$  are the diagonal, upper triangular, and lower triangular elements of  $\mathbf{A}$ , respectively. The spectral radius  $\mu$  is the largest eigenvalue of  $\mathbf{B}$ , which can be readily calculated [7].

Once calculated, the potential can be used to determine the electric and therefore the magnetic field of the TEM mode in the antenna.

## 2.2 Boundary Conditions

Boundary conditions were required at the edges of the mesh. For  $\theta = 0$  ( $i = 0$ ), the potential  $\Phi$  was zero, since the antenna was represented by a positively charged plate at  $\theta = +\theta_A$  and a negatively charged plate at  $\theta = -\theta_A$ . Therefore when  $i = 0$ ,

$$\mathbf{A}_{kk} = 1 \text{ and } \mathbf{A}_{k(k-1)} = \mathbf{A}_{k(k-1)} = \mathbf{A}_{k(k-j_{max})} = \mathbf{A}_{k(k+j_{max})} = 0$$

At  $\theta = \theta_A$  ( $i = i_{plate}$ ), the potential was set to  $V_0/2$ , where  $V_0$  is the total potential difference across the plates. Therefore for  $i = i_{plate}$ ,

$$\mathbf{A}_{kk} = 1 \text{ and } \mathbf{A}_{k(k-1)} = \mathbf{A}_{k(k-1)} = \mathbf{A}_{k(k-j_{max})} = \mathbf{A}_{k(k+j_{max})} = 0$$

and

$$\mathbf{b}_k = V_0/2$$

Since the TEM horn antenna is symmetric about the  $y$ - $z$  plane in figure 2.1,

$$\frac{\partial \Phi}{\partial \phi} = 0 \text{ at } \phi = \frac{\pi}{2} (j = 0) \text{ and } \phi = \frac{3\pi}{2} (j = N_\phi)$$

Hence  $\Phi(i,j-1) = \Phi(i,j+1)$ , which means that for  $j = 0$

$$\mathbf{A}_{k(k+1)} = \frac{1}{2} \Delta_\phi$$

and for  $j = N_\phi$

$$\mathbf{A}_{k(k-1)} = \frac{1}{2} \Delta_\phi$$

The final boundary condition was derived for the potential  $\theta = 0$  position at the top of the spherical mesh. At this point, the potential  $\Phi = \Phi_{top}$ . As  $\theta \rightarrow 0$ , Equation 1 infers that

$$\frac{\partial^2 \Phi}{\partial \phi^2} \rightarrow 0$$

If the potential in the vicinity of  $\theta = 0$  ( $i = N_\theta$ ) is represented in terms of  $\phi$  harmonics as

$$\Phi = \sum_{n=0}^{\infty} \Phi_n e^{jn\phi}$$

then

$$-n^2 \Phi_n \rightarrow 0$$

This implies that  $\Phi_n = 0$  near  $\theta = 0$  for  $n \neq 0$ . The  $n = 0$  component can be found by averaging the nearest neighbour values. From continuity of the electric field in free space, the  $n = 0$  component of  $\Phi$  at  $\theta = 0$  can be approximated by the value at  $\theta = \Delta_\theta$ , since

$$\frac{d\Phi_0}{d\theta} \rightarrow 0 \text{ as } \theta \rightarrow 0.$$

Therefore, for  $\theta = \Delta_\theta$  ( $i = N_\theta$ ) Equation 2 can be written as

$$\begin{aligned} \Phi(N_\theta-1,j) \frac{\sin \theta}{\Delta_\theta} \left[ \frac{\sin \theta}{\Delta_\theta} - \frac{\cos \theta}{2} \right] - 2 \Phi(N_\theta,j) \left[ \frac{\sin^2 \theta}{2} + \frac{1}{2} \right] + \Phi(N_\theta,j+1) \frac{1}{2} + \Phi(N_\theta,j-1) \frac{1}{2} \\ = -\Phi_{top} \frac{\sin \theta}{\Delta_\theta} \left[ \frac{\sin \theta}{\Delta_\theta} + \frac{\cos \theta}{2} \right] \end{aligned}$$

where

$$\Phi_{top} = \frac{1}{N_\phi+1} \sum_{j=0}^{N_\phi} \Phi(N_\theta,j)$$

Therefore, no modifications to  $A_{kl}$  need be made, and  $b_k$  is now given by

$$b_k = -\frac{\sin \theta}{\Delta_\theta} \left[ \frac{\sin \theta}{\Delta_\theta} + \frac{\cos \theta}{2} \right] \frac{1}{N_\phi+1} \sum_{j=0}^{N_\phi} \Phi(N_\theta,j)$$

for  $i = N_\theta$ .

### 2.3 Electric Field Calculation

The electric field quantities  $\alpha_\theta(i,j)$  and  $\alpha_\phi(i,j)$  which are related to the electric field through

$$E_\theta = V_0 \frac{\alpha_\theta(i,j)}{r} \quad \text{and} \quad E_\phi = V_0 \frac{\alpha_\phi(i,j)}{r}$$

are calculated in the centre of the finite difference mesh cells using the values of the potential at the vertices of the cells as follows

$$\begin{aligned} \alpha_\theta(i,j) &= -\frac{\partial \Phi}{\partial \theta} & = & \frac{\Phi(i,j+1) + \Phi(i,j) - \Phi(i+1,j) - \Phi(i+1,j+1)}{2 \Delta_\theta} \\ \alpha_\phi(i,j) &= -\frac{1}{\sin \theta} \frac{\partial \Phi}{\partial \phi} & = & \frac{\Phi(i,j) + \Phi(i+1,j) - \Phi(i,j+1) - \Phi(i+1,j+1)}{2 \sin \theta \Delta_\phi} \end{aligned}$$

$\alpha_\theta(\theta, \phi)$  and  $\alpha_\phi(\theta, \phi)$  can be found for the other three quadrants using

$$\alpha_\phi(\theta_i, \phi_j) = -\alpha_\phi(-\theta_i, \phi_j) = -\alpha_\phi(\theta_i, -\phi_j) = \alpha_\phi(-\theta_i, -\phi_j)$$

and

$$\alpha_\theta(\theta_i, \phi_j) = \alpha_\theta(-\theta_i, \phi_j) = \alpha_\theta(\theta_i, -\phi_j) = \alpha_\theta(-\theta_i, -\phi_j)$$

The electric field can now be calculated for the TEM mode of the antenna from the potential calculated on the finite difference mesh. The input impedance can now be calculated from the TEM mode field.

## 2.4 Input Impedance Calculation

The input impedance was determined by calculating the power,  $P$ , transmitted through a spherical surface enclosing the antenna as follows

$$P = \int_0^\pi \int_0^{2\pi} \frac{|\mathbf{E}|^2}{\zeta_0} a^2 \sin \theta d\phi d\theta$$

where  $|\mathbf{E}|$  is the magnitude of the electric field,  $\zeta_0$  is the impedance of free space, and  $a$  is the radius of the spherical surface. Using symmetry and substituting for the electric field,  $P$  can be expressed as

$$P = 4 \frac{V_0^2}{\zeta_0} \int_0^{\pi/2} \int_{\pi/2}^{3\pi/2} (\alpha_\theta^2 + \alpha_\phi^2) \sin \theta d\phi d\theta$$

The power can also be related to the voltage across the plates,  $V_0$ , and the antenna input impedance,  $Z_A$ , using

$$P = \frac{V_0^2}{Z_A}$$

Therefore

$$Z_A = \frac{\zeta_0}{4 \int_0^{\pi/2} \int_{\pi/2}^{3\pi/2} (\alpha_\theta^2 + \alpha_\phi^2) \sin \theta d\phi d\theta}$$

In most practical situations, the TEM horn will be fed by  $50 \Omega$  coax with an applied voltage  $V_{in}$ . If the antenna is matched to the feed cable, then  $V_0 = V_{in}$ . However, when there is a mismatch, the voltage on the antenna plates is related to the applied voltage by

$$V_0 = \frac{2 Z_A}{Z_A + Z_{coax}} V_{in}$$

where  $Z_{coax} = 50 \Omega$ .

The finite difference potential calculation therefore allows the input impedance of the TEM horn to be calculated from the TEM mode fields and from power considerations. The only approximations are due to the finite difference mesh discretisation and the small rounding of the antenna flare and apex angles,  $\theta_A$  and  $\phi_A$ , as described in Section 2.1.

### 3. Results

The antenna input impedance was measured for plate apex angles of  $20^\circ$ ,  $30^\circ$ , and  $45^\circ$  using a single triangular fin supported above a ground plane, with the antenna fed from underneath the ground plane via an N-type connector. The impedance of this configuration is half the impedance of a full horn. The impedance was determined for a range of flare angles,  $\theta_A$  from the reflection coefficient. The flare angle was limited to  $30^\circ$  due to mechanical constraints on the feed. Reflections from the aperture of the antenna were time-gated out so that the measurements could be related to infinitely long antennas. A Hewlett-Packard HP54120A 18 GHz digitising oscilloscope was used with an HP 54121A four-channel test set which possessed a time-domain reflectometry (TDR) capability. The reflection coefficient,  $\rho$ , for each antenna configuration was determined by measuring the amplitude of the reflected voltage,  $E_r$ , and dividing it by the height of the incident voltage step  $E_i$ , which was measured as 200 mV.

$$\rho = \frac{E_r}{E_i}$$

The error in  $\rho$  was calculated from the error in  $E_r$  and  $E_i$ ,  $\sigma(E)$ , using

$$\sigma(\rho) = \frac{\sqrt{1 + \rho^2}}{E_i} \sigma(E)$$

The error  $\sigma(E)$  was set to the cursor width of  $\pm 1.56$  mV.

The antenna input impedance was calculated from the reflection coefficient using

$$Z_A = Z_0 \frac{1 - \rho}{1 + \rho}$$

where  $Z_0 = 50 \Omega$  is the impedance of the oscilloscope input. The impedance error  $\sigma(Z_A)$  was calculated from the error in the reflection coefficient

$$\sigma(Z_A) = \frac{2 Z_A}{(1 - \rho)^2} \sigma(\rho)$$

Table 3.1 shows the measured input impedance  $Z_A^{exp}$  and the error,  $\sigma(Z_A^{exp})$  together with the theoretical results  $Z_A^{th}$  from the finite difference potential calculation, for various flare angles,  $\theta_A$ , for the  $20^\circ$  apex angle TEM horn. Tables 3.2 and 3.3 show the same parameters for  $30^\circ$  and  $45^\circ$  apex angle horns. These results are plotted in Figures 3.1, 3.2, and 3.3. The theoretical results are the continuous curves and the points correspond to measured impedance values. The agreement between the theoretical and experimental results is usually within experimental error. The calculated impedance was found to be systematically greater than the measured impedance for the  $45^\circ$  apex angle horn, as shown in Table 3.3 and Figure 3.3. This was probably due to the mechanical difficulties involved in rigidly supporting the large triangular plate above the ground plane.

The number of finite difference mesh points,  $N_\theta$  and  $N_\phi$ , were typically in the range 100-150. The tolerance for the overrelaxation scheme was  $10^{-5}$ . Under these conditions, the run time on a Sun SPARC 5 workstation was approximately 2-5 minutes for each flare and apex angle.

*Table 3.1: Measured and theoretical TEM horn input impedances for 20° apex angle*

$\theta_A$ (deg)	$Z_A^{exp}$ ( $\Omega$ )	$\sigma(Z_A^{exp})$ ( $\Omega$ )	$Z_A^{th}$ ( $\Omega$ )
5.739	76.6	1.2	75.4
6.892	85.5	1.4	86.9
8.048	96.9	1.5	94.6
9.207	104.8	1.6	106.1
10.37	113.3	1.8	114.9
11.54	122.6	1.9	123.3
12.71	128.6	2.1	131.4
13.89	137.0	2.2	136.13
15.07	144.6	2.4	143.8
16.26	151.0	2.5	151.0
17.46	156.0	2.6	157.1
18.66	161.2	2.7	162.3
19.88	169.5	2.9	166.3
21.10	172.3	3.0	171.9
22.33	178.3	3.1	177.1
23.58	184.4	3.3	182.1
24.83	187.6	3.4	186.5
26.10	194.2	3.5	189.5
27.39	197.7	3.6	193.6
28.69	201.2	3.7	197.5
30	204.8	3.8	201.4

*Table 3.2: Measured and theoretical TEM horn input impedances for 30° apex angle*

$\theta_A$ (deg)	$Z_A^{exp}$ ( $\Omega$ )	$\sigma(Z_A^{exp})$ ( $\Omega$ )	$Z_A^{th}$ ( $\Omega$ )
6.6	59.0	1.0	62.7
8.0	68.4	1.1	72.1
9.3	77.8	1.2	80.7
10.7	85.5	1.4	86.6
12.0	92.5	1.5	94.5
13.4	100.0	1.6	101.7
14.8	106.5	1.7	108.5
16.1	113.3	1.8	114.9
17.5	118.8	1.9	120.8
18.9	124.6	2.0	126.3
20.3	130.6	2.1	131.6
21.7	137.0	2.2	136.4
23.2	141.5	2.3	141.0
24.6	146.1	2.4	145.4
26.1	151.0	2.5	149.4
27.6	156.0	2.6	153.3
29.1	158.6	2.7	156.9
30.6	163.9	2.8	161.3
32.2	166.7	2.9	164.3

*Table 3.3: Measured and theoretical TEM horn input impedances for 45° apex angle*

$\theta_A$ (deg)	$Z_A^{exp}$ ( $\Omega$ )	$\sigma(Z_A^{exp})$ ( $\Omega$ )	$Z_A^{th}$ ( $\Omega$ )
5.7	35.5	0.8	38.7
6.9	40.7	0.8	45.6
8.0	46.3	0.9	50.5
9.2	51.5	0.9	56.2
10.4	57.1	1.00	62.7
11.5	61.0	1.04	67.6
12.7	66.2	1.1	72.5
13.9	71.8	1.2	76.8
15.0	76.6	1.2	81.2
16.3	80.3	1.3	85.6
17.5	84.2	1.3	89.8
18.7	88.2	1.4	93.9
19.9	92.5	1.5	97.3
21.1	96.9	1.5	101.0
22.3	100.0	1.6	104.5
23.6	103.2	1.6	107.7
24.8	108.1	1.7	110.8
26.1	111.6	1.8	114.1
27.4	115.1	1.8	116.8
28.7	117.0	1.8	119.8
30	120.7	1.9	122.6

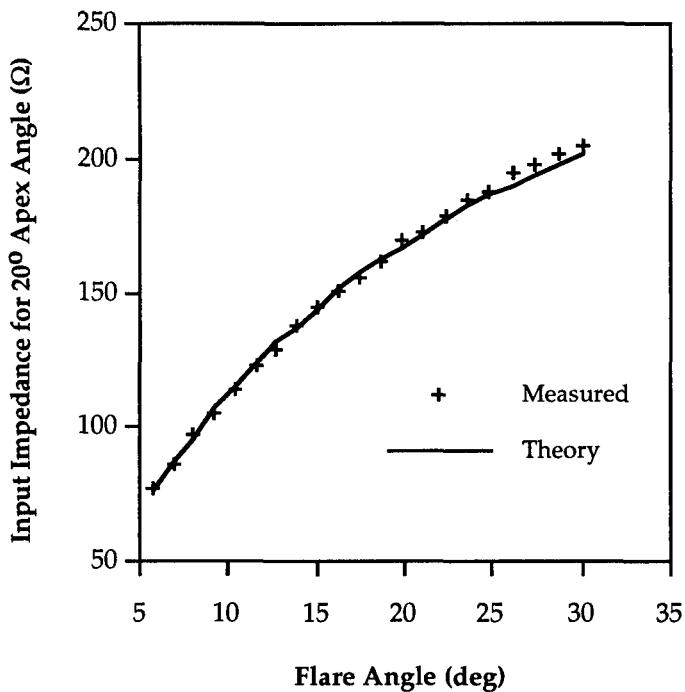


Figure 3.1: Input impedance against flare angle  $\theta_A$  for TEM horn with  $20^\circ$  apex angle

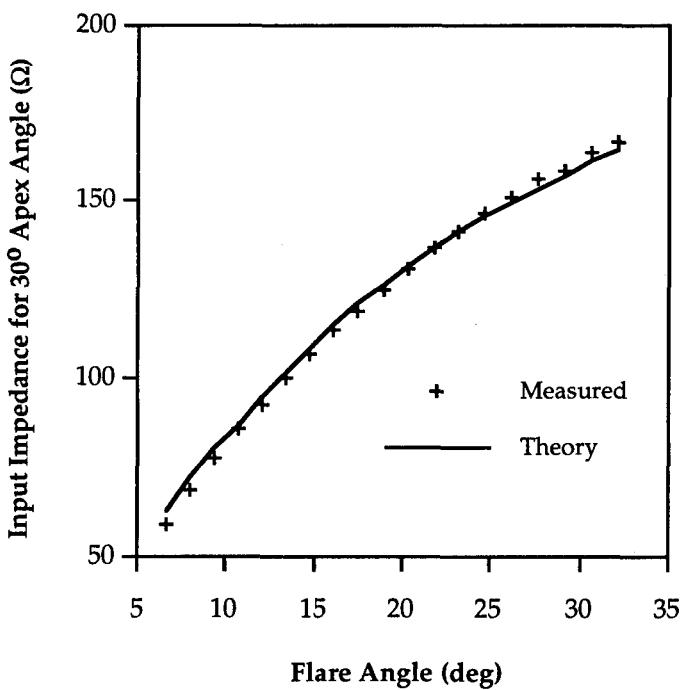


Figure 3.2: Input impedance against flare angle  $\theta_A$  for TEM horn with  $30^\circ$  apex angle

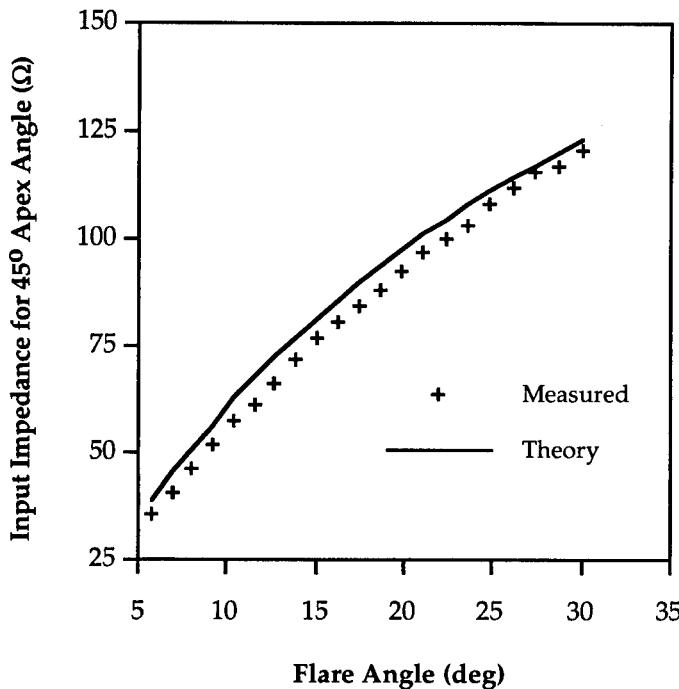


Figure 3.3: Input impedance against flare angle  $\theta_A$  for TEM horn with  $45^\circ$  apex angle

## 4. Conclusions

A relatively simple analysis has been presented which is useful for calculating the input impedance of TEM horn antennas suitable for radiating ultrawideband pulses. For ideal, infinitely-long antennas the input impedance is independent of frequency. This approximation is valid provided the length of the horn is much greater than the spatial pulse width. This is certainly the case for the TEM horns used in the DSTO Wideband Test Facility which are usually 1 m in length. A finite difference calculation in spherical geometry was used to calculate the TEM mode potential which could then be used to calculate the input impedance of the antenna with considerable accuracy. This approach is simpler than alternative conformal mapping techniques [2], [3] which until recently have been considered as unreliable. Experimental data agrees well with the theoretical predictions for the TEM horn input impedance. For plate apex angles of  $20^\circ$  and  $30^\circ$ , the agreement between the measured and predicted impedance values is generally better than a few percent. The discrepancy between theoretical and experimental results was somewhat worse for the horn with a  $45^\circ$  apex angle at approximately 10%. This was attributed to mechanical difficulties in supporting the plate. This analysis can now be used to simplify the

design of TEM horn antennas by allowing  $\theta_A$  to be calculated for a given  $\phi_A$  and excitation source output impedance.

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Input Impedance Calculation for  
TEM Horn Antennas

Geoff Staines and Sean Braidwood

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